Single photon reflection and transmission in optomechanical system

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Cavity Optomechanical system is speedily approaching the regime where the radiation pressure of a single photon displaces the moving mirror. In this paper, we consider a cavity optomechanical system where the cavity field is driven by an external field. In the limit of weak mirror-cavity couplings, we calculate analytically the reflection and transmission rates for cavity field and discuss the effects of mirror-cavity coupling on the reflection and transmission.

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I. INTRODUCTION

Cavity Optomechanics illustrates the radiation pressure by the interaction between an optical cavity mode and the motion of a mechanical object. The simplest example of such a system is a Fabry-Perot cavity with a moveable mirror. Optomechanics is a growing field of research studying the quantum dynamics of electromagnetic and mechanical degree of freedom coupled through radiation pressure and photothermal force or optical gradient[1–3]. In the fundamental optomechanical setup, the frequency of an optical cavity modulates parametrically with the position of mechanical oscillator. In most experiments, this optomechanical coupling is small compared to mechanical frequency and the linewidth of the cavity. However, if we drive an optical cavity strongly, then the cavity may contain a large number of photons, the coupling between cavity field and mechanical oscillator would be increased by a factor $\sqrt{p}$, where p is the mean number of photon in the cavity. Recently, this guides to observe the radiation-pressure effect, for example, normal mode splitting[4], optomechanically induced transparency[5, 6] and sideband colling[7, 8].

The interaction of light with matter tells us a great deal about the nature of the matter. It covers variety of applications in astrophysics, cosmology, quantum optics, and nanoscience. The coupling between the cavity field and the movable mirror in optomechanics has been a great attraction for researchers because they are helpful to create non-classical states of both cavity field and mirror [9], and it has also been used for quantum noise reduction[10]. Light-matter interactions can be produced efficiently by using optical cavities adjustment of mirrors that act as cavity resonators for light waves. Cavity quantum electrodynamic(QED) system is significant for examining light matter interactions. Coupling of a single two-level atom with a single mode of the electromagnetic field is reinforced by a cavity, which is important for the investigation of light-matter interaction[11–15].

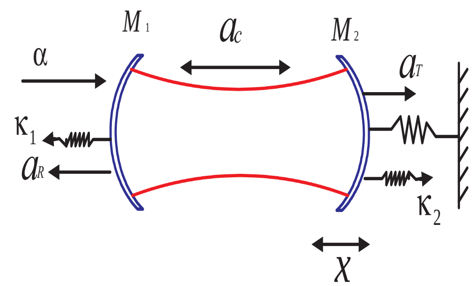

FIG. 1: Schematic description of an optomechanic setup, which consists of a mechanical oscillator coupled to a cavity by radiation pressure. α is the input field, a_c the cavity field, and a_R and a_T is the reflected and transmitted field, receptively. κ_1 and κ_2 denotes the cavity loss rate via the left and right mirror, respectively.

Jaynes-Cummings model is the basic model for atom-field interaction. This model consists of single mode radiation field coupled with two level atoms. Due to presence of strong coupling, the matter-field coupling increased the cavity field decay rate and atomic decay rate[16–18]. The coupled atom-cavity system can leads to a splitting in the atomic fluorescence spectrum and the empty-cavity transmission resonances. This splitting is known as vacuum-Rabi splitting[21, 23, 24]. Vacuum-Rabi splitting can be detected dynamically in population oscillation between two levels when field is resonant on the transition[22], as well as in fluorescence spectrum when the initial field strength is very small[23] and in a cavity transmission function profile at specific transition[24]. In this work, we calculate analytical reflection and transmission coefficients for field and intensities in the absence of phase noise to prevent the coherence of the cavity field.

This paper is arranged as follows. In section II, we describe the optomechanical system and its theoretical framework. In section III, we calculate steady state solution, leading reflection and transmission coefficient for the field as well as intensities. The findings are then discussed and presented in Section IV, while conclusion is given in section V.

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II. THEORETICAL FRAMEWORK

We consider an optomechanical cavity with frequency ω_c formed by a fixed end mirror M_1 and a moving end mirror M_2. The mirror M_2 can be considered as a harmonic oscillator with mass m and frequency ω_m. The oscillator mirror and the cavity are coupled with each other via radiation pressure. The system is coherently driven by the field $\alpha = \alpha e^{-i\omega_L t}$ with frequency ω_L as shown in figure 1. The cavity decay rate of mirrors are denoted by κ_1 and κ_2. In the rotating frame with driving frequency ω_L, the total Hamiltonian[12, 25–28] of the system can be written as

$$H = \hbar\Delta_c \hat{a}_c^\dagger \hat{a}_c + \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_m^2\hat{x}^2 - \hbar g \hat{a}_c^\dagger \hat{a}_c \hat{x} + i\hbar\sqrt{2\kappa_1}(\alpha\hat{a}_c^\dagger - \alpha^*\hat{a}_c), \tag{1}$$

Where $\Delta_c = \omega_c - \omega_L$ is the cavity resonance frequency detuning. $\hat{a}_c$ and $\hat{a}_c^\dagger$ are, respectively, the annihilation and creation operators for the cavity field of frequency ω_c with the commutation relation $[\hat{a}_c, \hat{a}_c^\dagger] = 1$. $\hat{p}$ and $\hat{x}$ are the momentum and displacement of the oscillating mirror with mass m and frequency ω_m. The parameter g is coupling strength of the radiation pressure between the mirror and the cavity, which is considered sufficiently weak in this paper.

The input-output theory for a light field interacting with a cavity is given by[29]

$$\begin{aligned} \hat{a}_R &= \sqrt{2\kappa_1}\hat{a}_c - \alpha \\ \hat{a}_T &= \sqrt{2\kappa_2}\hat{a}_c \end{aligned} \tag{2}$$

The first and second equations determine the reflected and transmitted field of the input and output mirror, using the cavity decay rate κ_1 and κ_2.

The dynamics of the the system is determined by using the master equation

$$\frac{\partial\rho}{\partial t} = \frac{1}{i\hbar}[H,\rho] + \mathcal{L}(\rho), \tag{3}$$

where $\mathcal{L}(\rho)$ is the Lindblad operator. This is used to model the incoherent decay processes and is given by

$$\mathcal{L}(\rho) = D\rho D^\dagger - \frac{1}{2}\rho D^\dagger D - \frac{1}{2}D^\dagger D\rho \tag{4}$$

The cavity mode is damped by photon leakage, which is designed by Lindblad term with $D = \sqrt{2\kappa}\hat{a}_c$, where κ is the total cavity-field decay rate and is given by $\kappa = \kappa_1 + \kappa_2$

By using the master equation, the dynamics of the system can be written as

$$\frac{\partial\langle\hat{a}_c\rangle}{\partial t} = -i\Delta_c\langle\hat{a}_c\rangle + ig\langle\hat{x}\rangle\langle\hat{a}_c\rangle + \sqrt{2\kappa_1}\alpha - \kappa\langle\hat{a}_c\rangle \tag{5}$$

$$\frac{\partial\langle\hat{a}_c^\dagger\hat{a}_c\rangle}{\partial t} = -2\kappa\langle\hat{a}_c^\dagger\hat{a}_c\rangle + \sqrt{2\kappa_1}(\alpha\langle\hat{a}_c^\dagger\rangle + \alpha^*\langle\hat{a}_c\rangle) \tag{6}$$

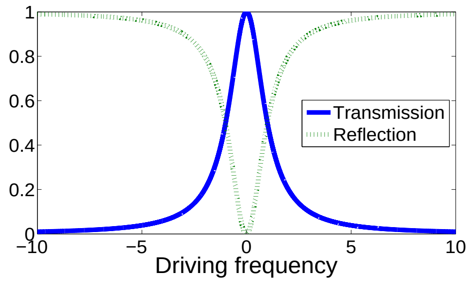

FIG. 2: The cavity-intensity-transmission and reflection coefficient as a function of driving frequency. Here $\kappa_1 = \kappa_2 = 0.5$, $m = 1$, $\omega_m = 1$, $\omega_c = 0$, $\gamma = 0$, $|\alpha|^2 = 1$, $g = 0.1$ and $\hbar = 1$.

$$\frac{\partial\langle\hat{x}\rangle}{\partial t} = \frac{1}{m}\langle\hat{p}\rangle \tag{7}$$

$$\frac{\partial\langle\hat{p}\rangle}{\partial t} = -m\omega_m^2\langle\hat{x}\rangle + \hbar g\langle\hat{a}_c^\dagger\hat{a}_c\rangle - \frac{\gamma}{m}\langle\hat{p}\rangle \tag{8}$$

Where γ is damping rate of the mechanical oscillator.

III. STEADY STATE SOLUTION

The steady state solution of equations (5-8) can be worked out by putting the time derivative equal to zero. The steady state solution of these equations leads to

$$\langle\hat{a}_c\rangle = \frac{\sqrt{2\kappa_1}\alpha}{\kappa + i(\Delta_c - g\langle\hat{x}\rangle)} \tag{9}$$

$$\langle\hat{a}_c^\dagger\hat{a}_c\rangle = |\langle\hat{a}_c\rangle|^2 \tag{10}$$

$$\langle\hat{p}\rangle = 0 \tag{11}$$

$$\langle\hat{x}\rangle = X_1 - X_2 + X_3. \tag{12}$$

Here $X_1 = \frac{2\Delta_c}{3g}$, $X_2 = \frac{2^{\frac{1}{3}}m_1}{3g^2m\omega_m^2(m_2+\sqrt{4m_1^3+m_2^2})^{\frac{1}{3}}}$, $X_3 = \frac{(m_2+\sqrt{4m_1^3+m_2^2})^{\frac{1}{3}}}{2^{\frac{1}{3}}3g^2m\omega_m^2}$, $m_1 = 3g^2\kappa^2m^2\omega_m^4 - g^2m^2\omega_m^4\Delta_c^2$ and $m_2 = 54g^5m^2\omega_m^4\alpha\hbar\alpha^*\kappa_1 - 18g^3\kappa^2m^3\omega_m^6\Delta_c - 2g^3m^3\omega_m^6\Delta_c^3$ $\langle\hat{a_c}\rangle$ and $\langle\hat{a_c}^\dagger\hat{a_c}\rangle$ are field amplitude and photon number. These results are very important for the manipulation of transmission and reflection intensity of the filed. The complex field-transmission and reflection coefficients can be written as

$$t = \frac{\langle\hat{a}_T\rangle}{\alpha} = \frac{2\sqrt{\kappa_1\kappa_2}}{\kappa + i(\Delta_c - g\langle\hat{x}\rangle)} \tag{13}$$

$$r = \frac{\langle\hat{a}_R\rangle}{\alpha} = \frac{2\kappa_1}{\kappa + i(\Delta_c - g\langle\hat{x}\rangle)} - 1 \tag{14}$$

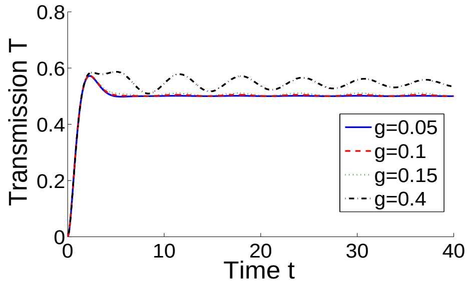

FIG. 3: Dynamics of the cavity-intensity-transmission coefficient T. Here $\kappa_1 = \kappa_2 = 0.5$, $m = 1$, $\omega_m = 1$, $\gamma = 0$, $\Delta_c = 1$, $|\alpha|^2 = 1$ and $\hbar = 1$. Initial condition for given curve is $\langle \hat{a}_c \rangle = \langle \hat{a}_c^\dagger \hat{a}_c \rangle = \langle \hat{p} \rangle = \langle \hat{x} \rangle = 0$

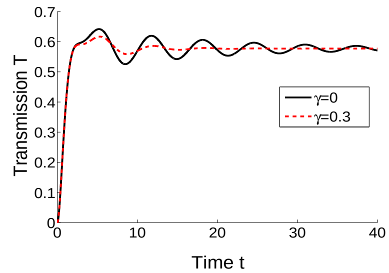

FIG. 4: Dynamics of the cavity-intensity-transmission coefficient with undamped and damped of the movable mirror. Here $\kappa_1 = \kappa_2 = 0.5$, $m = 1$, $\omega_m = 1$, $\Delta_c = 1$, $|\alpha|^2 = 1$, $g = 0.5$ and $\hbar = 1$. Initial condition for given curve is $\langle \hat{a}_c \rangle = \langle \hat{a}_c^\dagger \hat{a}_c \rangle = \langle \hat{p} \rangle = \langle \hat{x} \rangle = 0$

Intensity-reflection coefficient R,

$$R = \frac{\langle \hat{a}_R^\dagger \hat{a}_R \rangle}{|\alpha|^2}. \tag{15}$$

Intensity-transmission coefficient T,

$$T = \frac{\langle \hat{a}_T^\dagger \hat{a}_T \rangle}{|\alpha|^2} \tag{16}$$

By using equation (2), we can find R and T.

$$R = |r|^2 = \frac{4\kappa_1^2}{\kappa^2 + (\Delta_c - g\langle \hat{x} \rangle)^2} - \frac{2\kappa_1}{\kappa + i(\Delta_c - g\langle \hat{x} \rangle)} - \frac{2\kappa_1}{\kappa - i(\Delta_c - g\langle \hat{x} \rangle)} + 1 \tag{17}$$

$$T = |t|^2 = \frac{4\kappa_1\kappa_2}{\kappa^2 + (\Delta_c - g\langle \hat{x} \rangle)^2} \tag{18}$$

Here r and t, respectively, are the coefficient of reflection and transmission for the fields given by equation (13,14).

IV. RESULTS AND DISCUSSIONS

Equations (17) and (18) are the main result of the present paper. The sum of R and T is equal to 1. The cavity intensity transmission and reflection coefficient as a function of driving frequency are shown in Fig.2. When detuning is zero, the transmission rate arrives at its maximum. As we increased or decreased the detuning, the transmission rate becomes smaller and smaller. In our work, we use an approximation, i.e., $\langle \hat{x}\hat{a}_c \rangle = \langle \hat{x} \rangle \langle \hat{a}_c \rangle$.

The condition which ensures the validity of the weak coupling approximation can be derived from Eq.(5-8). To make the derivation clear, we ignore the damping of the harmonic oscillator, i.e., $\gamma = 0$. The weak coupling condition can be found by examining the linear dependence of $\langle \hat{a}_c \rangle$ on the coupling constant g. For this purpose, we ignore the term with g in equation (8) and have $\frac{\partial^2 \langle \hat{x} \rangle}{\partial t^2} = -\omega_m^2 \langle \hat{x} \rangle$, leading to $\langle \hat{x} \rangle = x_0 \sin(\omega_m t + \phi)$, where $x_0 = \langle \hat{x}(t = 0) \rangle$ is the maximum amplitude of oscillation and ϕ is the initial phase angle. Substituting this result into equation (5), we have

$$\frac{\partial \langle \hat{a}_c \rangle}{\partial t} = -i\Delta_c \langle \hat{a}_c \rangle + ig\langle \hat{a}_c \rangle x_0 \sin(\omega_m t + \phi) + \sqrt{2\kappa_1}\alpha - \kappa \langle \hat{a}_c \rangle. \tag{19}$$

The resulting $\langle \hat{a}_c \rangle$ clearly is a function of time t and g, i.e., $\langle \hat{a}_c \rangle = \langle \hat{a}_c(g, t) \rangle$. To find how $\langle \hat{a}_c(g, t) \rangle$ linearly depends on g, we expand $\langle \hat{a}_c \rangle$ as follows,

$$\langle \hat{a}_c(g, t) \rangle = \langle \hat{a}_c(0, t) \rangle + g\langle \hat{a}_{c1}(t) \rangle + g^2 \langle \hat{a}_{c2}(t) \rangle + ... \tag{20}$$

Substituting this expansion into Eq. (19), we find for the zeroth order of g,

$$\frac{\partial \langle \hat{a}_c(0, t) \rangle}{\partial t} = -i\Delta_c \langle \hat{a}_c \rangle + \sqrt{2\kappa_1}\alpha - \kappa \langle \hat{a}_c \rangle \tag{21}$$

For first order of g, we have

$$\frac{\partial \langle \hat{a}_{c1}(t) \rangle}{\partial t} = (-i\Delta_c - \kappa)\langle \hat{a}_{c1}(t) \rangle + i\langle \hat{a}_c(0, t) \rangle x_0 \sin(\omega_m t + \phi) \tag{22}$$

After taking integration of equation (22) and putting the result into equation (20), we have

$$\langle \hat{a}_c(g, t) \rangle = \langle \hat{a}_c(0, t) \rangle + \frac{g x_0}{2\omega_m} \langle \hat{a}_c(0, t) \rangle \sin(\omega_m t + \phi) \tag{23}$$

Clearly, the weak coupling limit means,

$$\eta = \frac{g x_0}{2\omega_m} \ll 1 \tag{24}$$

It is the condition for the validity of weak coupling regime. Fig.3 shows the dynamics of the intensity-transmission coefficient T as a function of time. For small value of coupling constant, our approximation is valid,i.e. the transmission can reach a steady value as time approaches infinity. As we increased the value of

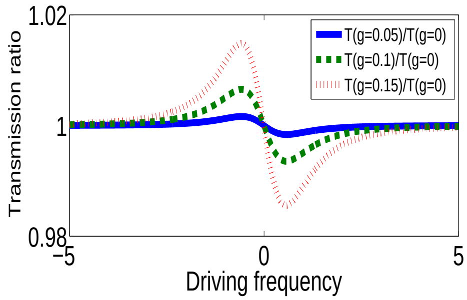

FIG. 5: The cavity-intensity-transmission ratio $\frac{T(g\neq 0)}{T(g=0)}$) as a function of driving frequency ω_L. For given curves, $\kappa_1=\kappa_2=0.5$, $m=1$, $\omega_m=1$, $\omega_c=0$, $\gamma=0$, $|\alpha|^2=1$ and $\hbar=1$.

coupling constant from $g=0.1$ to $g=0.4$, there is a large oscillation in transmission T, which means our approximation is not valid for strong couplings. However, If we use damping of movable mirror, then we may consider the strong coupling, because the damping term speed up damping and our system comes to a stationary state very quickly as shown in Fig.4. For steady state result, we plot the ratio of transmission rate at $g=0$ to that of $g\neq 0$, i.e. $\frac{T(g\neq 0)}{T(g=0)}$), see Fig.5. It shows that, for large detuning, transmission rates are independent of coupling constant and their ratio ia almost 1. For small detuning, however, the coupling constant has very large effect on the transmission rate. The larger the value of coupling constant, the larger the transmission rate is. For positive detuning, i.e. $\Delta_c>0$, transmission rate with coupling is larger than the transmission rate without coupling $T(g\neq 0)>T(g=0)$. For negative detuning, i.e. $\Delta_c<0$, transmission rate with coupling is smaller than the transmission rate without coupling,i.e. $T(g\neq 0)<T(g=0)$.

V. CONCLUSIONS

In this paper, we study the transmission and reflection rates of an optical cavity, which has a moving mirror and is driven coherently by an external field. We aim at the effects of the moving mirror on the transmission and reflection. Our analysis is based on the weak mechanical interaction. We calculated analytically the steady state transmission and reflection coefficients as a function of detuning Δ_c and of the cavity-mirror coupling constant. In addition, we show that for small value of coupling constant g, our approximation is valid, but it fails to treat the case of strong couplings.

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